

Exam 4 – Review Sheet

Sections: 17.1 - 18.2

In addition to solving the following problems,
make sure you have reviewed all the problems from class and homework's.

1. Differentiate: a) $y = \cos(5x^3)$ g) $y = \frac{e^{2x}}{\sin(2x)}$ b) $y = \tan^4(2x)$ h) $y = \cos(x^2 - 9x)$ c) $y = \sin^2 x + \cos^2 x$ i) $y = \cos^4(x) + \cos(t^4)$ d) $y = \frac{\cos(x^2)}{e^{2x}}$ e) $y = \sin x \cos x$ f) $y = e^{13x} \cos x$	2. Integrate: a) $\int x \cos(x^2) dx$ g) $\int 3 \cos(2\pi x) dx$ b) $\int \tan^3 x \sec^2 x dx$ h) $\int e^{2x} \sin(e^{2x}) dx$ c) $\int \cot x dx$ d) $\int \sin^7 x \cos x dx$ e) $\int \tan^2 x dx$ (use a Pythagorean Id.) f) $\int (x - 1) \cos(x^2 - 2x) dx$
3. a) Find the area under the graph of $y = \sin x$ over the interval $[0, \pi]$ b) Evaluate: $\int_0^{\frac{\pi}{2}} \cos(4x) dx$ c) Evaluate: $\int_0^{\frac{\pi}{3}} \sec^2(x) dx$ d) Evaluate: $\int_0^3 (50 + 25 \sin(\frac{2\pi}{3}x)) dx$	4. Differential Equations: a) Let $y' - 2y = 0$, show that: i) $y = e^{2x}$ is a particular/exact solution. ii) $y = Ce^{2x}$ is a general solution. What is the value of C for the above question i) b) Let $y'' + 6y' - 16y = 0$, show that: i) $y = C_1 e^{-8x} + C_2 e^{2x}$ is a general solution. ii) Provide 3 particular/exact solutions to i)
5. Separable Differential Equations: a) Solve $2\sqrt{y} \frac{dy}{dx} - 3x = 0$ if $x = 0$ when $y = 9$ b) Solve $y' = x^2 y$ c) Solve $(1 + x^2)y' = xy$ d) Solve $y' = 4x^3 + 2x + 5$ if $x = 1$ when $y = 10$ e) Solve $\frac{dy}{dx} = 3x + xy$ if $x = 0$ when $y = 2$	6. Find the demand function $q = D(x)$ if the Elasticity of Demand is: (Recall: $E(x) = -\frac{x D'(x)}{D(x)}$) a) $E(x) = 2$ b) $E(x) = x$, if $q = 2$ when $x = 4$ c) $E(x) = \frac{4}{x}$, if $q = 2$ when $x = 4$

Solutions

1. a) $y' = -15x^2 \sin(5x^3)$ b) $y' = 8 \tan^3(2x) \sec^2(2x)$ c) $y' = 0$ d) $y' = \frac{-2(x \sin x^2 + \cos x^2)}{e^{2x}}$ e) $y' = \cos^2 x - \sin^2 x$ f) $y' = 13e^{13x} \cos(x) - \sin(x) e^{13x}$ g) $y' = \frac{2e^{2x}(\sin 2x - \cos 2x)}{\sin^2 2x}$ h) $y' = -(2x - 9)\sin(x^2 - 9x)$ i) $y' = -4 \sin t \cdot \cos^3 t - 4t^3 \sin(t^4)$	2. a) $\frac{1}{2} \sin x^2 + C$ b) $\frac{1}{4} \tan^4 x + C$ c) $\ln \sin x + C$ d) $\frac{\sin^8 x}{8} + C$ e) $\tan x - x + C$ f) $\frac{1}{2} \sin(x^2 - 2x) + C$ g) $\frac{3}{2\pi} \sin(2\pi x) + C$ h) $-\frac{1}{2} \cos(e^{2x}) + C$
3. a) 2 b) 0 c) $\sqrt{3}$ d) 150	4. a) i) see notes ii) see notes, $C = 1$ b) i) see notes ii) $y = -e^{-8x} + 3e^{2x}$, $y = 4e^{-8x} - 6e^{2x}$, $y = \pi e^{-8x} + \frac{11}{3}e^{2x}$ (answers may vary)
5. a) $y = \left(\frac{9}{8}x^2 + 27\right)^{\frac{2}{3}}$ b) $y = Ce^{\frac{x^3}{3}}$ c) $y = C\sqrt{1+x^2}$ d) $y = x^4 + x^2 + 5x + 3$ e) $y = 5e^{\frac{x^2}{2}} - 3$	6. a) $q = \frac{C}{x^2}$ b) $q = Ce^{-x}$, $C = 2e^4$, $q = 2e^{4-x}$ c) $q = Ce^{\frac{4}{x}}$, $C = \frac{2}{e}$, $q = 2e^{\frac{4}{x}-1}$ or $q = 2e^{-1+\frac{4}{x}}$